

On atmospheric neutrino mass-squared difference in the precision era

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Abstract

With the measurement of the small parameter $\sin^2\theta_{13}$ experiments on the study of neutrino oscillations enter into a high precision era. I discuss here the problem of the definition of the atmospheric mass-squared difference which will be important for analysis of data of future experiments.

Discovery of neutrino oscillations driven by small neutrino mass-squared differences and neutrino mixing is one of the most important recent discovery in the particle physics. Small neutrino masses is an evidence of a new scale in physics, presumably much larger than the electroweak scale.

First neutrino oscillation data were interpreted as the two-neutrino $\nu_\mu \rightleftharpoons \nu_\tau$ oscillations in the atmospheric range of L/E and $\bar{\nu}_e \rightleftharpoons \bar{\nu}_{\mu,\tau}$ oscillations in the solar (KamLAND) range of L/E (see [1]). These oscillations were described by four parameters: Δm_{23}^2 and $\sin^2 2\theta_{23}$ (the atmospheric and accelerator long baseline experiments) and Δm_{12}^2 and $\sin^2 2\theta_{12}$ (the reactor KamLAND experiment).

With measurement of the small parameter $\sin^2 2\theta_{13}$ in the T2K [2], Daya Bay [3], RENO [4] and Double CHOOZE [5] experiments the study of neutrino oscillations enter into a new era, the era of the high precision measurements. At this stage a % effects of the three-neutrino mixing are planned to be revealed.

In the case of the three-neutrino mixing we have

$$\nu_{lL} = \sum_{i=1}^3 U_{li} \nu_{iL}. \quad l = e, \mu, \tau \quad (1)$$

Here ν_{lL} is the flavor neutrino field, ν_i is the field of neutrino with mass m_i . The unitary PMNS mixing matrix U is characterized by three mixing angles and one CP phase and in the standard parametrization has the form

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{13}c_{23} \end{pmatrix}, \quad (2)$$

where $c_{12} = \cos \theta_{12}$, $s_{12} = \sin \theta_{12}$ etc.

Usually, in accordance with the solar neutrino data, neutrino masses are labeled in such a way that $m_2 > m_1$ and $\Delta m_{12}^2 \equiv \Delta m_S^2 > 0$ is the solar mass-squared difference.¹ In the case of the three neutrinos two neutrino mass spectra are possible:

1. Normal spectrum (NS) : Δm_{12}^2 is the difference between square of masses of the lightest neutrinos: $m_3 > m_2 > m_1$
2. Inverted spectrum (IS): Δm_{12}^2 is the difference between square of masses of the heaviest neutrinos: $m_3 < m_1 < m_2$.

Accuracies of the existing neutrino oscillation data do not allow to establish the character of the neutrino mass spectrum. It is one of the major problem of the future high precision neutrino oscillation experiments.

In the case of the three-neutrino mixing neutrino transition probabilities depend on sixth oscillation parameters. In all analysis of the neutrino oscillation data parameters $\theta_{12}, \theta_{23}, \theta_{13}, \delta$ and Δm_S^2 , which are determined in the same way for the normal and inverted neutrino mass spectra, are used. We will discuss now the problem of the choice of atmospheric mass-squared difference, the sixths neutrino oscillation parameter.

In literature exist *different definitions of the atmospheric neutrino mass-squared difference*.

1. In [6, 2] this parameter is determined as a modulus of a difference of square of the mass of ν_3 and square of the mass of the "intermediate" neutrino (ν_2 in NS case and ν_1 in IS case):

$$\Delta m_A^2 = \Delta m_{23}^2 \text{ (NS)}, \quad \Delta m_A^2 = |\Delta m_{13}^2| \text{ (IS)}. \quad (3)$$

2. The Bari group [7] determines the atmospheric mass-squared difference as follows

$$(\Delta m_A^2)_B = \frac{1}{2} |\Delta m_{13}^2 + \Delta m_{23}^2| \text{ (NS, IS)} \quad (4)$$

3. The NuFit group [8] determines the atmospheric mass-squared difference in the following way

$$(\Delta m_A^2)_{NF} = \Delta m_{13}^2 \text{ (NS)} \quad (\Delta m_A^2)_{NF} = |\Delta m_{23}^2| \text{ (IS)} \quad (5)$$

¹The mass-squared difference Δm_{ki}^2 is determined as follows $\Delta m_{ki}^2 = m_i^2 - m_k^2$.

4. In analysis of the data of Daya Bay [3] and RENO [4] experiments the following "large" mass-squared difference was used

$$\Delta m_{ee}^2 = \cos^2 \theta_{12} \Delta m_{13}^2 + \sin^2 \theta_{12} \Delta m_{23}^2 \quad (6)$$

Let us notice that with the definition given in 1. vacuum neutrino transition probabilities have the simple form of the sum of atmospheric, solar and interference terms [6]

$$\begin{aligned} P^{NS}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) &= \delta_{ll'} - 4|U_{l3}|^2(\delta_{ll'} - |U_{l'3}|^2) \sin^2 \Delta_A \\ &- 4|U_{l1}|^2(\delta_{ll'} - |U_{l'1}|^2) \sin^2 \Delta_S - 8 [\text{Re} (U_{l'3} U_{l3}^* U_{l'1}^* U_{l1}) \cos(\Delta_A + \Delta_S) \\ &\pm \text{Im} (U_{l'3} U_{l3}^* U_{l'1}^* U_{l1}) \sin(\Delta_A + \Delta_S)] \sin \Delta_A \sin \Delta_S, \end{aligned} \quad (7)$$

and

$$\begin{aligned} P^{IS}(\bar{\nu}_l \rightarrow \bar{\nu}_{l'}) &= \delta_{ll'} - 4|U_{l3}|^2(\delta_{ll'} - |U_{l'3}|^2) \sin^2 \Delta_A \\ &- 4|U_{l2}|^2(\delta_{ll'} - |U_{l'2}|^2) \sin^2 \Delta_S - 8 [\text{Re} (U_{l'3} U_{l3}^* U_{l'2}^* U_{l2}) \cos(\Delta_A + \Delta_S) \\ &\mp \text{Im} (U_{l'3} U_{l3}^* U_{l'2}^* U_{l2}) \sin(\Delta_A + \Delta_S)] \sin \Delta_A \sin \Delta_S. \end{aligned} \quad (8)$$

Here $\Delta_{A,S} = \frac{\Delta m_{A,S}^2 L}{4E}$, where L is the detector-source distance and E is the neutrino energy.

It is obvious that parameters $\Delta m_A^2, (\Delta m_A^2)_B, (\Delta m_A^2)_{NF}$ are determined in the same way for normal and inverted spectra. We have

$$(\Delta m_A^2)_B = \Delta m_A^2 + \frac{1}{2} \Delta m_S^2, \quad (\Delta m_A^2)_{NF} = \Delta m_A^2 + \Delta m_S^2. \quad (9)$$

From analysis of neutrino oscillation data it follows that $\frac{\Delta m_S^2}{\Delta m_A^2} \simeq 3 \cdot 10^{-2}$. Thus, different definitions of the atmospheric mass-squared difference differ by a few %. However, the goal of future neutrino oscillation experiments is to measure oscillation parameters with a % accuracy. In the precision era *one definition of atmospheric mass-squared difference will be definitely important*. Theoretically there is no preferred definition. From our point of view a consensus must be found what definition is the most suitable from the practical point of view.

The "averaged mass-squared difference"

$$\Delta m_{ll}^2 = \cos^2 \theta_{ll} \Delta m_{13}^2 + \sin^2 \theta_{ll} \Delta m_{23}^2, \quad l = e, \mu \quad (10)$$

was introduced in [9]. Here

$$\cos^2 \theta_{ll} = \frac{|U_{l1}|^2}{|U_{l1}|^2 + |U_{l2}|^2}, \quad \sin^2 \theta_{ll} = \frac{|U_{l2}|^2}{|U_{l1}|^2 + |U_{l2}|^2}. \quad (11)$$

The probability of $\bar{\nu}_l^{(-)}$ to survive in vacuum can be presented in the form

$$P(\bar{\nu}_l^{(-)} \rightarrow \bar{\nu}_l^{(-)}) = 1 - 4|U_{l1}|^2|U_{l2}|^2 \sin^2 \Delta_{12} - 4|U_{l3}|^2(1 - |U_{l3}|^2)(\cos^2 \theta_{ll} \sin^2 \Delta_{13} + \sin^2 \theta_{ll} \sin^2 \Delta_{23}), \quad (12)$$

where $\Delta_{ki} = \frac{\Delta m_{ki}^2 L}{4E}$. We have $\Delta m_{13}^2 = \Delta m_{23}^2 + \Delta m_{12}^2$. Taking into account this relation from (10) we find

$$\Delta m_{13}^2 = \Delta m_{ll}^2 + \sin^2 \theta_{ll} \Delta m_{12}^2, \quad \Delta m_{23}^2 = \Delta m_{ll}^2 - \cos^2 \theta_{ll} \Delta m_{12}^2. \quad (13)$$

In reactor and long baseline accelerator experiments $\Delta_{12} \ll 1$ and $|\Delta_{13}| \simeq |\Delta_{23}| \simeq 1$. It is easy to see that in the expansion of the term $(\cos^2 \theta_{ll} \sin^2 \Delta_{13} + \sin^2 \theta_{ll} \sin^2 \Delta_{23})$ over Δ_{12} the linear term disappear. If we neglect the quadratic term, for $\bar{\nu}_e \rightarrow \bar{\nu}_e$ transition probability from (12) we find

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_S - \sin^2 2\theta_{13} \sin^2 \Delta_{ee}. \quad (14)$$

This expression was used for the analysis of the latest Daya Bay [3] and RENO [4] data. We would like now to comment the usage of the parameters Δm_{ee}^2 and $\Delta m_{\mu\mu}^2$.

- These parameters describe data of only disappearance experiments.
- Their definition depends on the character of neutrino mass spectrum. In fact, we have

$$\Delta m_{ee}^2 = \Delta m_A^2 + \cos^2 \theta_{12} \Delta m_S^2 \text{ (NS)}, \quad |\Delta m_{ee}^2| = \Delta m_A^2 + \sin^2 \theta_{12} \Delta m_S^2 \text{ (IS)}. \quad (15)$$

- In order to determine the fundamental parameter Δm_A^2 from the measured value of $|\Delta m_{ee}^2|$ and compare the reactor, atmospheric and accelerator data we need to know $\sin^2 \theta_{12}$, Δm_S^2 and also the neutrino mass spectrum.

Convenient alternative expressions the $\bar{\nu}_e$ survival probability in the case of the normal and inverted neutrino mass spectra, which follow from (7) and (8), have the form

$$\begin{aligned} P^{\text{NS}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) &= 1 - \sin^2 2\theta_{13} \sin^2 \Delta_A \\ &\quad - (\cos^4 \theta_{13} \sin^2 2\theta_{12} + \cos^2 \theta_{12} \sin^2 2\theta_{13}) \sin^2 \Delta_S \\ &\quad - 2 \sin^2 2\theta_{13} \cos^2 \theta_{12} \cos(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S. \end{aligned} \quad (16)$$

and

$$\begin{aligned} P^{\text{IS}}(\bar{\nu}_e \rightarrow \bar{\nu}_e) &= 1 - \sin^2 2\theta_{13} \sin^2 \Delta_A \\ &\quad - (\cos^4 \theta_{13} \sin^2 2\theta_{12} + \sin^2 \theta_{12} \sin^2 2\theta_{13}) \sin^2 \Delta_S \\ &\quad - 2 \sin^2 2\theta_{13} \sin^2 \theta_{12} \cos(\Delta_A + \Delta_S) \sin \Delta_A \sin \Delta_S. \end{aligned} \quad (17)$$

We believe that in the era of the high precision neutrino oscillation experiments, *data must be analyzed in terms of universal fundamental neutrino oscillation parameters* (mixing angles, phase and independent mass-squared differences) which characterize all transition probabilities and are directly connected with neutrino mixing matrix and masses.

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